AQA

A-LEVEL
Mathematics

MM05 - Mechanics 5
Mark scheme

6360

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Copyright © 2018 AQA and its licensors. All rights reserved.
AQA retains the copyright on all its publications. However, registered schools/colleges for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Q	Solution	Mark	Total	Comment
1	$T = 2\pi\sqrt{\frac{l}{g}}$	B1		B1: Quoting formula for period.
	$1.05 \times 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{L}{g}}$	M1		M1: Increasing length by 5%. A1: Correct length or ratio.
	$L = 1.05^2 l = 1.1025l$	A1		A1: Correct percentage increase in length.
	10.25% increase needed	A1	4	
	Total		**4**	

Q	Solution	Mark	Total	Comment
2(a)	$a\omega = 0.16 \Rightarrow a = \dfrac{4}{25\omega}$	**B1**		B1: Using $a\omega = 0.16$ M1: Substituting into SHM formula. A1: Correct equation. bM1: Use of their ω to find period. A1: Correct period.
	$\dfrac{3}{125} = \omega^2(a^2 - 0.02^2)$	**M1** **A1**		
	$\dfrac{3}{125} = \omega^2\left(\dfrac{16}{625\omega^2} - \dfrac{1}{2500}\right)$	**dM1**		
	$60 = 64 - \omega^2$			
	$\omega = 2$			
	Period $= \dfrac{2\pi}{2} = \pi$ seconds	**A1**	**5**	
(b)	$m\dfrac{d^2x}{dt^2} = -k(x+e) + mg$			M1: Forming an equation to find *k.* A1: Correct equation. M1: Solving for *k*. A1: Correct *k*.
	But $e = \dfrac{mg}{k}$	**M1A1**		
	$m\dfrac{d^2x}{dt^2} = -kx - mg + mg$			
	$m\dfrac{d^2x}{dt^2} = -kx$	**M1**		
	$\omega^2 = \dfrac{k}{m}$			
	$k = 2^2 \times 1.5 = 6$ N m^{-1}	**A1**	**4**	
(c)	$a = \dfrac{0.16}{2} = 0.08$	**B1**		B1: Correct amplitude. M1: Equation to find extension at equilibrium. A1: Correct extension. M1: Equation to give max tension. A1: Correct maximum tension.
	At equilibrium position: $1.5 \times 9.8 = 6e$	**M1** **A1F**		
	$e = 2.45$			
	At maximum extension: $T = 6(2.45 + 0.08)$	**M1**		
	$= 15.2$ N	**A1**	**5**	
	Total		**14**	

Q	Solution	Mark	Total	Comment
3(a)	$V_A = 2mg \times 4\cos\theta$	**B1**		B1: Correct GPE for *A*.
	$V_B = -3mg \times 3\sin\theta$	**B1**		B1: Correct GPE for *B*.
	$V_C = 5mg \times 4\sin\left(\frac{\pi}{3} - \theta\right)$	**M1**		M1: Attempts GPE for *C*.
	$= 20mg\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)$	**A1**		A1: Correct GPE for *C*.
	$V = 8mg\cos\theta - 9mg\sin\theta + 10\sqrt{3}mg\cos\theta - 10mg\sin\theta$			A1: Results
	$= mg\left(\left(8 + 10\sqrt{3}\right)\cos\theta - 19\sin\theta\right)$	**A1**	**5**	combined and simplified to required result.
(b)	$\frac{dV}{d\theta} = mg\left(-\left(8 + 10\sqrt{3}\right)\sin\theta - 19\cos\theta\right)$	**M1**		M1: Differentiates *V*.
	$= mg\left(-\left(8 + 10\sqrt{3}\right)\sin\theta - 19\cos\theta\right)$	**A1**		A1: Correct derivative.
	$\frac{dV}{d\theta} = 0$			M1: Finds value for tan θ.
	$\tan\theta = \frac{-19}{8 + 10\sqrt{3}}$	**M1**		A1: Correct angles. Condone 143° and 323°
	$\theta = 2.50 \quad \text{or} \quad 5.64$	**A1**	**4**	
(c)	$\frac{d^2V}{d\theta^2} = mg\left(-\left(8 + 10\sqrt{3}\right)\cos\theta + 19\sin\theta\right)$	**M1**		M1: Obtains second derivative.
	$\theta = 2.50$			M1: Substitutes and considers both values.
	$\frac{d^2V}{d\theta^2} = 31.7mg > 0$	**M1**		A1: One correct conclusion.
	$\therefore$ Stable	**A1**		A1: Second correct conclusion.
	$\theta = 5.64$			
	$\frac{d^2V}{d\theta^2} = -31.7mg < 0$			Accept rigorously
	$\therefore$ Unstable	**A1**	**4**	reasoned alternatives.
	Total		**13**	

Q	Solution	Mark	Total	Comment
4(a)	$\frac{20}{0.5}e = 1.6 \times 9.8$	**M1**		M1: Equation to find extension.
	$e = 0.392$	**A1**		A1: Correct extension.
	Length = 0.892 m	**A1**	**3**	A1: Includes 0.5.
(b)(i)	$1.6\frac{d^2x}{dt^2} = 1.6 \times 9.8 - T$	**M1**		M1: Equation of motion involving *mg* and *T*.
	$= 15.68 - \frac{20}{0.5}(x - 0.1\sin(10t) - 0.5)$	**M1** **A1**		M1: Attempts expression for tension.
	$\frac{d^2x}{dt^2} = 9.8 - 25x + 2.5\sin(10t) + 12.5$			
	$\frac{d^2x}{dt^2} + 25x = 22.3 + 2.5\sin(10t)$	**A1**	**4**	A1: Correct tension. A1: Required result from correct working.
(b)(ii)	CF			M1: Roots of aux equation.
	$\lambda^2 + 25 = 0$	**M1**		A1: Correct form of CF.
	$\lambda = \pm 5i$			
	$x = A\cos(5t) + B\sin(5t)$	**A1**		
	PI			M1: Correct form of PI.
	$x = C\cos(10t) + D\sin(10t) + E$	**M1**		A1: Correct derivatives.
	$\dot{x} = -10C\sin(10t) + 10D\cos(10t)$			
	$\ddot{x} = -100C\cos(10t) - 100D\sin(10t)$	**A1**		M1: Substitution to find constants.
	$-100C\cos(10t) - 100D\sin(10t) +$ $25(C\cos(10t) + D\sin(10t) + E) = 22.3 + 2.5\sin(10t)$	**M1**		A1: Correct values of *E* and *C*.
	$E = \frac{22.3}{25} = 0.892, C = 0$	**A1**		A1: Correct *D*.
	$-100D + 25D = 2.5$	**A1**		A1: Correct PI.
	$D = -\frac{2.5}{75} = -\frac{1}{30}$			
	$x = 0.892 - \frac{1}{30}\sin(10t)$	**A1**		

	$x = A\cos(5t) + B\sin(5t) + 0.892 - \frac{1}{30}\sin(10t)$ $x = 0.892, t = 0$ $0.892 = A + 0.892$ $A = 0$ $\dot{x} = 5B\cos(5t) - \frac{1}{3}\cos(10t)$ $\dot{x} = 0, t = 0$ $0 = 5B - \frac{1}{3}$ $B = \frac{1}{15}$ $x = \frac{1}{15}\sin(5t) + 0.892 - \frac{1}{30}\sin(10t)$	**M1** **M1** **A1** **A1**	**12**	M1: Equation to find *A*. M1: Equation to find *B*. A1: Correct *A* and *B*. A1: Correct expression for *x*.
	Total		**19**	

Q	Solution	Mark	Total	Comment
5(a)	$\dot{\theta} = \frac{2}{5}$	**B1**		B1Correct $\dot{\theta}$
	$\dot{r} = \cos\theta\dot{\theta} = \frac{2}{5}\cos\theta$	**M1**		M1: Expression for $\dot{r}$.
	$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$	**M1**		M1: Finds v^2
	$= \frac{4}{25}\cos^2\theta + \frac{4}{25}\left(1 + 2\sin\theta + \sin^2\theta\right)$			A1: Correct v^2
	$= \frac{8}{25}\left(1 + \sin\theta\right)$	**A1**		A1: Correct constant and conclusion.
	$v = \frac{2\sqrt{2}}{5}\sqrt{r}$	**A1**	**5**	
	$\therefore$ Speed proportional to $\sqrt{r}$			
(b)	$\ddot{r} = -\frac{2}{5}\sin\theta\dot{\theta} = -\frac{4}{25}\sin\theta$			M1: Attempts both components. A1: One correct component. A1: Second correct component.
	$\ddot{r} - r\dot{\theta}^2 = -\frac{4}{25}\sin\theta - (1+\sin\theta)\times\frac{4}{25}$	**M1**		
	$= -\frac{4}{25}(1 + 2\sin\theta)$	**A1**		
	$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{8}{25}\cos\theta$	**A1**		
	$a^2 = \frac{16}{625}(1 + 4\sin\theta + 4\sin^2\theta) + \frac{64}{625}\cos^2\theta$	**M1**		M1: Expression for a^2 A1: Correct magnitude of acceleration. A1: Correct min. A1: Correct max.
	$= \frac{16}{625}(5 + 4\sin\theta)$			
	$a = \frac{4}{25}\sqrt{5 + 4\sin\theta}$	**A1**		
	$a_{\max} = \frac{12}{25} = 0.48$	**A1**		
	$a_{\min} = \frac{4}{25} = 0.16$	**A1**	**7**	
	Total		**12**	

Q	Solution	Mark	Total	Comment
6(a)	$\frac{dr}{dt} = kr$			B1: Correct constant of proportionality.
	$0.002 = 0.001k$	**B1**		M1: Attempts impulse-momentum equation.
	$k = 2$			A1: Correct equation.
	$g(m+\delta m)\delta t = (m+\delta m)(v+\delta v) - mv$	**M1**		
	$gm\delta t + g\delta m\delta t = m\delta v + v\delta m + \delta m\delta v$			
	$mg = m\frac{dv}{dt} + v\frac{dm}{dt}$	**A1**		
				M1: Expression for mass.
	$m = \frac{4}{3}\pi r^3\rho$	**M1**		
				M1: Derivative of m with respect to *t*.
	$\frac{dm}{dt} = 4\pi r^2\rho\frac{dr}{dt}$	**M1**		
		A1		A1: Obtaining 6*m*.
	$= 8\pi r^3\rho$			
	$= 6m$			A1: Required result from correct working.
	$mg = m\frac{dv}{dt} + 6mv$	**A1**	**7**	
	$\frac{dv}{dt} = g - 6v$			
(b)	$\int \frac{1}{g-6v}dv = \int 1\,dt$	**M1**		M1: Separation of variables.
	$-\frac{1}{6}\ln(g-6v) = t + c$	**A1**		A1: Correct integrals
	$g - 6v = Ae^{-6t}$	**M1**		M1: Finding constant of integration.
	$t = 0, v = U$	**A1**		A1: Correct constant.
	$A = g - 6U$			
	$v = \frac{g-(g-6U)e^{-6t}}{6}$	**A1**	**5**	A1: Correct expression for *v*.
(c)	$V \to \frac{g}{6}$	**B1**	**1**	B1: Correct limit.
	Total		**13**	